On the Analysis of Symmetrical Three-Line Microstrip Circuits

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Abstract—**The immittance parameters for the case of symmetrical coupled three-line microstrip or other inhomogeneous six-port structures are derived in terms of the normal modes of the coupled system. The analytical results obtained reduce to the heretofore known results when the line parameters are interrelated in a specified manner, and should be useful in the study and accurate design of three-line couplers and other microwave circuit elements.**

I. Introduction

MULTIPLE coupled line structures in an inhomogeneous medium such as microstrip lines may be used for applications as couplers [1], [2] and other multisection circuit elements. Symmetrical three-line structures (Fig. 1) have been analyzed for the homogeneous medium (TEM) case [3]. Attempts have also been made to analyze the inhomogeneous structures primarily for applications as couplers by assuming a set of modes [1], [2] which, as shown in this paper, for the three-line case are not generally the normal uncoupled wave modes of the system and hence cannot propagate independently. The general solutions for normal mode propagation constants, eigenvectors, and impedances, etc., are available in matrix form for the general case of multiconductor systems, e.g., [4], [5]. However, explicit solutions for the normal mode properties and six-port circuit parameters for the symmetrical three-line case in terms of the self- and mutual-line impedances and admittances per unit length are desirable and convenient to study and formulate design procedures for various applications as couplers and other circuit elements. These six-port circuit parameters are derived in this paper for the general case of symmetrical coupled three-line systems, and it is shown that the results obtained reduce to the known results when the line parameters are interrelated in a specified manner for the respective cases.

II. Analysis of Symmetrical Three-Line Structures

The voltages and currents on the three lines are given by the transmission line equations

$$\frac{d[V]}{dx} = -[z]\ [I] \tag{1a}$$

$$\frac{d[I]}{dx} = -[y]\ [V] \tag{1b}$$

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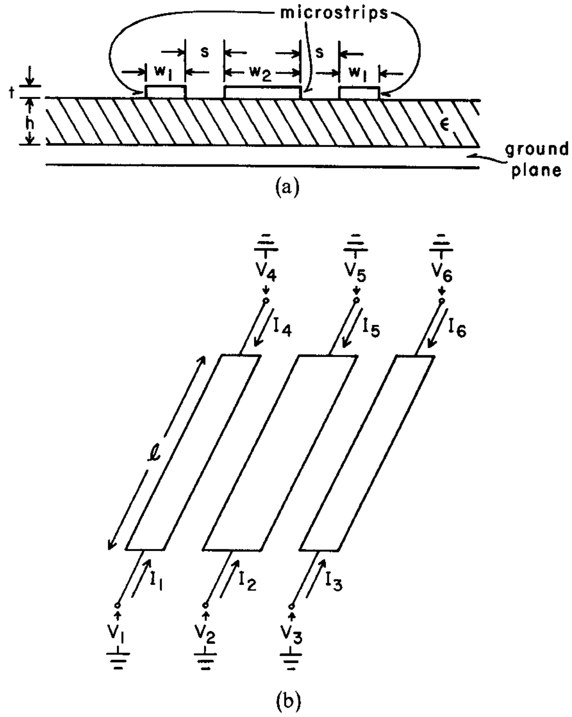

Fig. 1. (a) Cross sectional view of the symmetrical three-line microstrip structure. (b) Schematic of the coupled line six-port.

where, $[V]$ and $[I]$ are the three-dimensional column vectors and $[z]$ and $[y]$ are 3×3 impedance and admittance matrices. For the symmetric case, these are given by

$$[z] = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{12} & z_{22} & z_{12} \\ z_{13} & z_{12} & z_{11} \end{bmatrix}$$

$$[y] = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{22} & y_{12} \\ y_{13} & y_{12} & y_{11} \end{bmatrix}. \tag{2}$$

The voltages and currents for the case of uniformly coupled lines considered here are then the solution of

$$\frac{d^2[V]}{dx^2} + [z]\ [y]\ [V] = 0 \tag{3a}$$

and

$$\frac{d^2[I]}{dx^2} + [y]\ [z]\ [I] = 0. \tag{3b}$$

The characteristic product matrices $[z]\ [y]$ and $[y]\ [z]$ are of the form

$$[z]\ [y] = \begin{bmatrix} A & B & C \\ D & E & D \\ C & B & A \end{bmatrix} \tag{4a}$$

and

$$[y]\ [z] = [[z]\ [y]]^T \tag{4b}$$

where

$$A = z_{11}y_{11} + z_{12}y_{12} + z_{13}y_{13}$$
$$B = z_{11}y_{12} + z_{12}y_{22} + z_{13}y_{12}$$
$$C = z_{11}y_{13} + z_{12}y_{12} + z_{13}y_{11}$$
$$D = z_{12}y_{11} + z_{22}y_{12} + z_{12}y_{13}$$

and

$$E = z_{22}y_{22} + 2z_{12}y_{12}. \tag{5}$$

The eigenvalues which are the solutions of $|[z] \times [y] - \gamma^2[I]| = 0$ lead to the propagation constants for the normal modes of the system and are given by

$$\gamma_a^2 = A - C$$
$$\gamma_b^2 = \frac{A+C+E}{2} + 1/2\sqrt{(A+C-E)^2 + 8DB}$$
$$\gamma_c^2 = \frac{A+C+E}{2} - 1/2\sqrt{(A+C-E)^2 + 8DB}. \tag{6}$$

The corresponding voltage and current eigenvector matrices corresponding to these eigenvalues are given by

$$[M_V] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & R_{V1} & R_{V2} \\ -1 & 1 & 1 \end{bmatrix}$$

and

$$[M_I] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & R_{I1} & R_{I2} \\ -1 & 1 & 1 \end{bmatrix} \tag{7}$$

where

$$R_{V1,2} = -\frac{A+C-E}{2B} \pm \sqrt{\left(\frac{A+C-E}{2B}\right)^2 + 2\frac{D}{B}} \tag{8}$$

and

$$R_{I1,2} = -\frac{2}{R_{V2,1}}. \tag{9}$$

From (8) and (9) it is seen that

$$R_{V1}R_{V2} = -2\frac{D}{B}$$
$$R_{I1}R_{I2} = -2\frac{B}{D}. \tag{10}$$

Note that $R_{V1,2} \neq R_{I1,2}$ unless $D = B$. That is, in general, the ratio of voltages on the lines is not equal to the ratio of the currents for the normal modes of propagation and an *a priori* assumption that $R_{V1,2} = R_{I1,2}$ or $[M_V] = [M_I]$ for symmetrical multiconductor systems does not generally lead to the normal independent modes. The voltage and current eigenvectors which give the ratio of voltages and currents, respectively, on the three lines for the three normal modes of propagation may be used to find the characteristic impedances and admittances of the three lines for the three normal modes. From (1a) and (1b), these are found to be

$$Z_{a1} = Z_{a3} = \frac{z_{11} - z_{13}}{\gamma_a} = \frac{\gamma_a}{y_{11} - y_{13}} = \frac{1}{Y_{a1}} = \frac{1}{Y_{a3}} \tag{11a}$$

$$Z_{b1} = Z_{b3} = \frac{z_{11} + z_{13} + R_{I1}z_{12}}{\gamma_b} = \frac{\gamma_b}{y_{11} + y_{13} + R_{V1}y_{12}} = \frac{1}{Y_{b1}} = \frac{1}{Y_{b3}} \tag{11b}$$

$$Z_{b2} = \frac{R_{I1}z_{22} + 2z_{12}}{R_{I1}\gamma_b} = \frac{R_{V1}\gamma_b}{R_{V1}y_{22} + 2y_{12}} = \frac{1}{Y_{b2}} \tag{11c}$$

$$Z_{c1} = Z_{c3} = \frac{z_{11} + z_{13} + R_{I2}z_{12}}{\gamma_c} = \frac{\gamma_c}{y_{11} + y_{13} + R_{V2}y_{12}} = \frac{1}{Y_{c1}} = \frac{1}{Y_{c3}} \tag{11d}$$

$$Z_{c2} = \frac{R_{I2}z_{22} + 2z_{12}}{R_{I2}\gamma_c} = \frac{R_{V2}\gamma_c}{R_{V2}y_{22} + 2y_{12}} = \frac{1}{Y_{c2}} \tag{11e}$$

where Z_{jk} and Y_{jk} ($j = a,b,c$, and $k = 1,2,3$) are the characteristic impedance and admittance, respectively, of line k for mode j. From (8)–(10), it is seen that the characteristic immittances of the lines for the three normal modes are related through

$$\frac{Y_{b1}}{Y_{b2}} = \frac{Y_{c1}}{Y_{c2}} = \frac{Z_{b2}}{Z_{b1}} = \frac{Z_{c2}}{Z_{c1}} = -\frac{R_{V1}R_{V2}}{2}. \tag{12}$$

The immittance matrix parameters of the coupled line six-port (Fig. 1) are found in terms of these normal mode characteristic immittances of the three lines, the mode voltage and current ratios as given by (7)–(9), and the propagation constants for the normal modes $\gamma_{a,b,c}$ as given by (6). This is done in a straightforward manner, as for the case of the coupled line four-port [6], by writing the equations for voltages and currents for the six-ports in terms of the normal modes of the system. That is, $[V] = [C_V]\ [A]$ where $[V]$ and $[A]$ are six-dimensional column vectors and $[C_V]$ is the 6×6 matrix representing the relationship of the port voltages $V_{1..6}$ to the amplitudes of the forward and reflected waves $A_{1..6}$ for the three normal modes. The port currents are also related to these amplitude coefficients since for each mode j the current on line k is related to the voltage by $I_k = Y_{jk}V_k$, i.e., $[I] = [C_I]\ [A]$. Eliminating the amplitude coefficients leads to the impedance and admittance matrices for the coupled line six-port whose elements are

$$Z_{11} = Z_{33} = Z_{44} = Z_{66} = \tfrac{1}{2}[Z_{a1} \coth \gamma_a l - (R_{V2} Z_{b1} \coth \gamma_b l - R_{V1} Z_{c1} \coth \gamma_c l)/R_d] \quad (13a)$$

$$Z_{12} = Z_{21} = Z_{23} = Z_{32} = Z_{45} = Z_{54} = Z_{56} = Z_{65} = [Z_{b2} \coth \gamma_b l - Z_{c2} \coth \gamma_c l]/R_d \quad (13b)$$

$$Z_{13} = Z_{31} = Z_{46} = Z_{64} = -\tfrac{1}{2}[Z_{a1} \coth \gamma_a l + (R_{V2} Z_{b1} \coth \gamma_b l - R_{V1} Z_{c1} \coth \gamma_c l)/R_d] \quad (13c)$$

$$Z_{14} = Z_{41} = Z_{36} = Z_{63} = \tfrac{1}{2}[Z_{a1} \operatorname{csch} \gamma_a l + (-R_{V2} Z_{b1} \operatorname{csch} \gamma_b l + R_{V1} Z_{c1} \operatorname{csch} \gamma_c l)/R_d] \quad (13d)$$

$$Z_{15} = Z_{51} = Z_{24} = Z_{42} = Z_{35} = Z_{53} = Z_{26} = Z_{62} = [Z_{b2} \operatorname{csch} \gamma_b l - Z_{c2} \operatorname{csch} \gamma_c l]/R_d \quad (13e)$$

$$Z_{16} = Z_{61} = Z_{34} = Z_{43} = -\tfrac{1}{2}[Z_{a1} \operatorname{csch} \gamma_a l + (R_{V2} Z_{b1} \operatorname{csch} \gamma_b l - R_{V1} Z_{c1} \operatorname{csch} \gamma_c l)/R_d] \quad (13f)$$

$$Z_{22} = Z_{55} = [R_{V1} Z_{b2} \coth \gamma_b l - R_{V2} Z_{c2} \coth \gamma_c l]/R_d \quad (13g)$$

$$Z_{25} = Z_{52} = [R_{V1} Z_{b2} \operatorname{csch} \gamma_b l - R_{V2} Z_{c2} \operatorname{csch} \gamma_c l]/R_d \quad (13h)$$

$$Y_{11} = Y_{33} = Y_{44} = Y_{66} = \tfrac{1}{2}[Y_{a1} \coth \gamma_a l - (R_{V2} Y_{b1} \coth \gamma_b l - R_{V1} Y_{c1} \coth \gamma_c l)/R_d] \quad (14a)$$

$$Y_{12} = Y_{21} = Y_{23} = Y_{32} = Y_{45} = Y_{54} = Y_{56} = Y_{65} = [Y_{b1} \coth \gamma_b l - Y_{c1} \coth \gamma_c l]/R_d \quad (14b)$$

$$Y_{13} = Y_{31} = Y_{46} = Y_{64} = -\tfrac{1}{2}[Y_{a1} \coth \gamma_a l + (R_{V2} Y_{b1} \coth \gamma_b l - R_{V1} Y_{c1} \coth \gamma_c l)/R_d] \quad (14c)$$

$$Y_{14} = Y_{41} = Y_{36} = Y_{63} = -\tfrac{1}{2}[Y_{a1} \operatorname{csch} \gamma_a l + (-R_{V2} Y_{b1} \operatorname{csch} \gamma_b l + R_{V1} Y_{c1} \operatorname{csch} \gamma_c l)/R_d] \quad (14d)$$

$$Y_{15} = Y_{51} = Y_{24} = Y_{42} = Y_{35} = Y_{53} = Y_{26} = Y_{62} = -[Y_{b1} \operatorname{csch} \gamma_b l - Y_{c1} \operatorname{csch} \gamma_c l]/R_d \quad (14e)$$

$$Y_{16} = Y_{61} = Y_{34} = Y_{43} = \tfrac{1}{2}[Y_{a1} \operatorname{csch} \gamma_a l + (R_{V2} Y_{b1} \operatorname{csch} \gamma_b l - R_{V1} Y_{c1} \operatorname{csch} \gamma_c l)/R_d] \quad (14f)$$

$$Y_{22} = Y_{55} = [R_{V1} Y_{b2} \coth \gamma_b l - R_{V2} Y_{c2} \coth \gamma_c l]/R_d \quad (14g)$$

$$Y_{25} = Y_{52} = -[R_{V1} Y_{b2} \operatorname{csch} \gamma_b l - R_{V2} Y_{c2} \operatorname{csch} \gamma_c l]/R_d \quad (14h)$$

where $R_d \triangleq (R_{V1} - R_{V2})$ and l is the length of the lines.

The above formulation may be used to study and evaluate the properties of any symmetrical three-line system in terms of the equivalent self- and mutual-series impedances and shunt admittances per unit length of the lines. For the case of lossless lines $\gamma_{a,b,c} l = j\beta_{a,b,c} l$ and the hyperbolic functions may be replaced by trigonometric functions, i.e., $\coth \gamma_{a,b,c} l = -j \cot \theta_{a,b,c}$ and $\operatorname{csch} \gamma_{a,b,c} l = -j \csc \theta_{a,b,c}$, where $\theta_{a,b,c} = \beta_{a,b,c} l$. $\theta_{a,b,c}$, the electrical length of the lines for the three normal modes, are linearly dependent on frequency for quasi-TEM case of coupled microstrip lines at low frequencies. For this case, the results can then be expressed in terms of the capacitance matrix for the structure and the capacitance matrix of the same structure with dielectric removed.

The analytical results obtained above reduce to known results for coupled two-line case [6], when one of the three lines is removed, and to the known results for three-line cases [2], [3] when the line parameters are interrelated in a specified manner. For the case of the homogeneous medium, the eigenvalues are degenerate and $\gamma_a = \gamma_b = \gamma_c = j\omega\sqrt{\mu\varepsilon}$ leads to the results obtained by Yamamoto *et al.* [3] where a convenient set of eigenvector matrices can be chosen to study the coupled line six-port. For the inhomogeneous medium lossless quasi-TEM case, e.g., coupled microstrip lines at low frequencies, the results obtained above reduce to those obtained by Pavlidis and Hartnagel [2] iff $D = B$. It is seen that for $[M_V] = [M_I]$ which is an implicit assumption in the previous formulations, $D = B$ for the three line case. Then from (8)–(10)

$$R_{V1,2} = R_{I1,2} \qquad R_{V1} R_{V2} = R_{I1} R_{I2} = -2$$

$$Z_{b1} = Z_{b2} \quad \text{and} \quad Z_{c1} = Z_{c2} \quad (15)$$

and the results obtained for characteristic impedances and six-port parameters reduce to those obtained in [2], provided the correct expression for R_{V1} as given by (8) is utilized. For this case, if the coupling between nonadjacent lines is neglected ($y_{13} = z_{13} = 0$), the condition $D = B$ requires that [from (5)]

$$\frac{z_{12}}{z_{11} - z_{22}} = \frac{y_{12}}{y_{11} - y_{22}}. \quad (16)$$

This condition is obviously satisfied when $z_{11} = z_{22}$ and $y_{11} = y_{22}$. For this case the solution for the propagation constants, the mode voltage ratios and the characteristic impedances as given by (6), (8), and (11) for the three normal modes are found to be

$$R_{V1} = -R_{V2} = \sqrt{2} \quad (17)$$

$$\gamma_a = \sqrt{z_{11} y_{11}} \quad (18a)$$

$$\gamma_b = \sqrt{(z_{11} + \sqrt{2} z_{12})(y_{11} + \sqrt{2} y_{12})} \quad (18b)$$

$$\gamma_c = \sqrt{(z_{11} - \sqrt{2} z_{12})(y_{11} - \sqrt{2} y_{12})} \quad (18c)$$

$$Z_{a1} = Z_{a3} = \sqrt{z_{11}/y_{11}} \tag{19a}$$

$$Z_{b1} = Z_{b2} = Z_{b3} = \sqrt{(z_{11} + \sqrt{2}\, z_{12})/(y_{11} + \sqrt{2}\, y_{12})} \tag{19b}$$

$$Z_{c1} = Z_{c2} = Z_{c3} = \sqrt{(z_{11} - \sqrt{2}\, z_{12})/(y_{11} - \sqrt{2}\, y_{12})}. \tag{19c}$$

The experimental results given in [2] agree well with the theoretical predictions since the conditions given above are approximately satisfied for the experimental structure which consists of identical relatively loosely coupled microstrip lines. However, for the general case of three-line microstrip structures the analytical results given in this paper can be utilized to formulate design procedures for couplers and other circuit elements.

III. Conclusions

Symmetrical three-line microstrip or other inhomogeneous structures may be studied in terms of the normal uncoupled wave modes and six-port circuit parameters of the system. The expressions for the propagation constants, characteristic impedances, and six-port parameters have been derived explicitly in terms of the distributed self- and mutual- impedances and admittances per unit length of the lines. For the case of coupled microstrip lines, the results obtained should be useful in the study and design of couplers and other circuit elements. The theory applies to any symmetrical coupled three-line dispersive, lossy, passive, or reciprocal-active system and may be used to study such systems in terms of their equivalent self- and mutual-line parameters.

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Intermodulation Distortion Analysis of Reflection-Type IMPATT Amplifiers Using Volterra Series Representation

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***Abstract*—Intermodulation distortion generated in a stable IMPATT amplifier is analyzed using Volterra series representation. An IMPATT amplifier model, which takes into account the interaction between the nonlinearities of the diode and its embedding circuitry, is described. The Volterra transfer functions are derived for this model. Nonlinear terms up to and including the fifth order are considered. Intermodulation distortion products are calculated for a low-level input signal consisting of two tones. The results of this analysis are extrapolated into the direction of increasing output power in order to obtain the third-order intercept point. Further, closed form expressions for the third-order intermodulation IM_3 and intercept point P_I are derived. The distortion of a specific 6-GHz IMPATT amplifier is evaluated for illustrative purposes; the predicted distortion behavior compares favorably with experimental results.**

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I. Introduction

Microwave oscillators and amplifiers using IMPATT diodes are being utilized in areas such as telecommunications [1]. In many such applications, the intermodulation noise arising from IMPATT diode nonlinearities becomes an important consideration. This paper investigates the nonlinear distortion produced in an IMPATT amplifier, using Volterra series as an analysis tool. The Volterra series expansion allows a detailed and accurate